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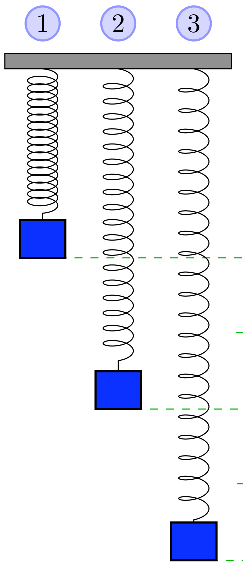
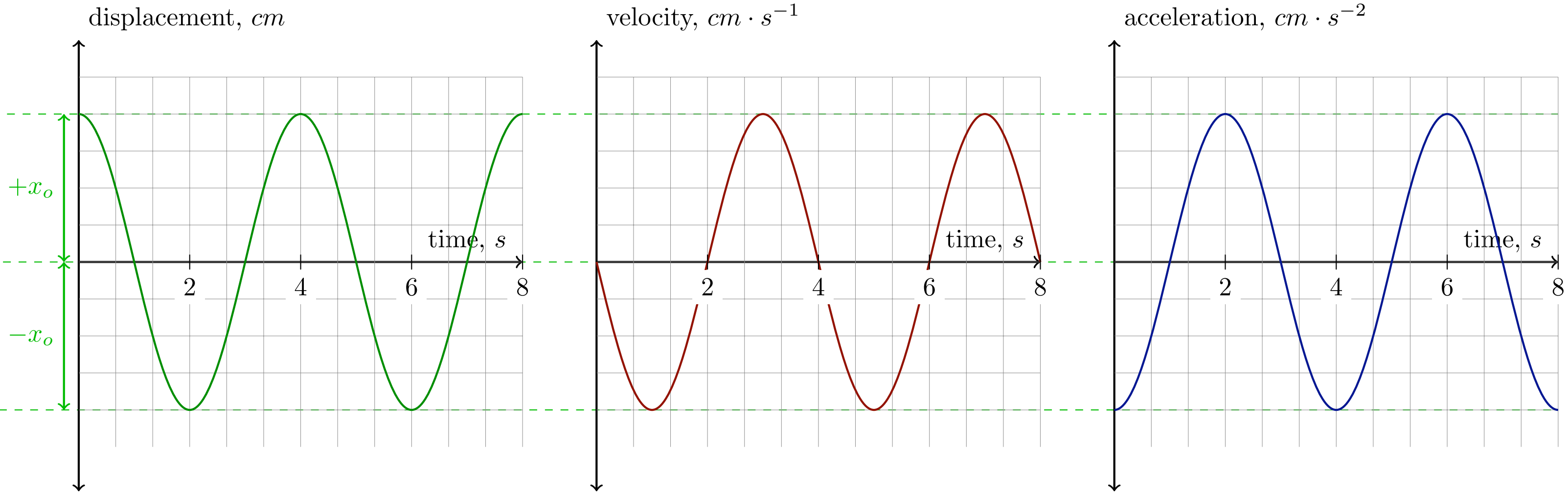
SHM Kinematics 1 (4.1.5)



1 = maximum compression point, $+x_o$
2 = equilibrium point, 0
3 = maximum expansion point, $-x_o$



•Angular frequency, ω, measures the (# of revolutions) per (1 second)
• Remember that $2\pi f = \omega$
•We denote the maximum displacement as x_o (at 0,2,4,6,8 seconds)
• Thus the equation of this line is:
$\mathbf{x = x_o cos\omega t}$

•From the gradient of the displacement/time graph, we can calculate the velocity
•We denote the maximum velocity as v_o (at 1,3,5,7 seconds)
• Thus the equation of this line is:
$\mathbf{v = -v_o sin\omega t}$

•From the gradient of the velocity/time graph, we can calculate the acceleration
•We denote the maximum acceleration as a_o (at 1,3,5,7 seconds)
• Thus the equation of this line is:
$\mathbf{a = -a_o cos\omega t}$

• When displacement increases, acceleration increases proportionally and negatively:
$\mathbf{a \propto -x}$

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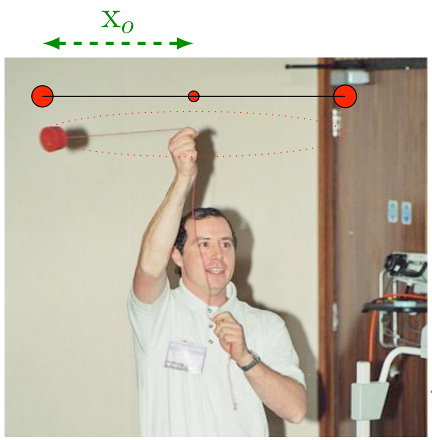
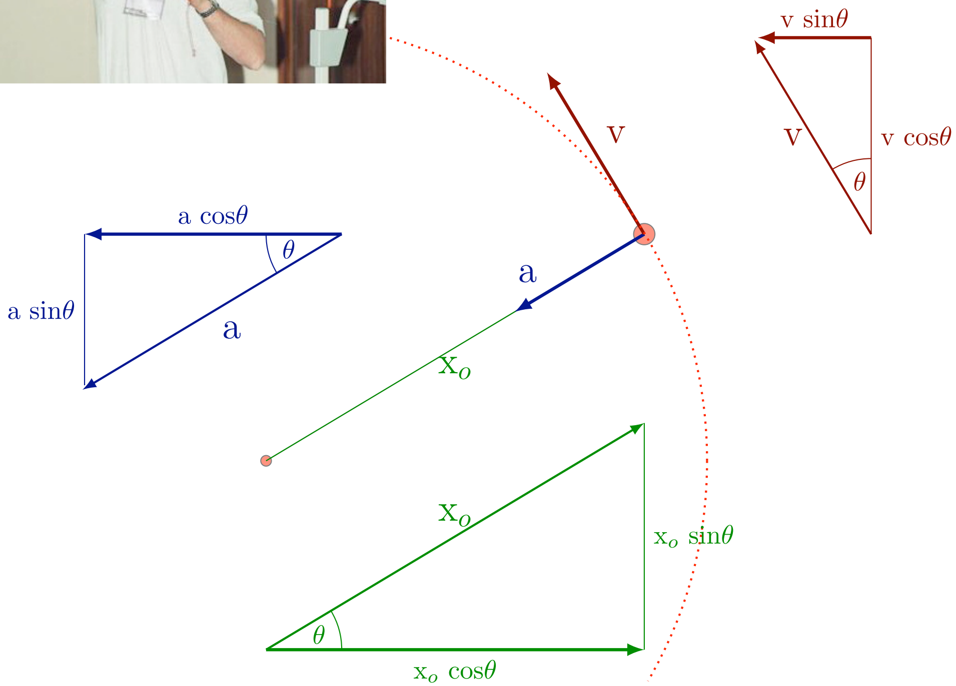
SHM Kinematics 2 (4.1.3)



• When viewed from above, the red ball would supposedly be moving in a circle
• When viewed from straight ahead, if you disregard depth perception, the red ball would supposedly be moving horizontally back and forth, in SHM. We are going to be focusing on the **horizontal SHM motion**.
• If a body oscillating with SHM has an angular frequency of $\omega = 2\pi$f and amplitude x_o then it's displacement(x), velocity (v) and acceleration (a) at any given time can be derived from these formulas:



Velocity

1) Velocity $= \dfrac{\text{distance}}{\text{time}} = \dfrac{2\pi \text{r}}{\text{t}} = \omega$r
2) If $v = \omega r$, here we have $v = \omega x_o$
3) The horizontal velocity at displacement x is $v = -v sin\theta$
4) Add 2 and 3 we get horizontal velocity $= -\omega x_o sin\theta$

Acceleration

1) Centripetal acceleration $= \dfrac{\text{v}^2}{\text{r}} = \dfrac{\omega^2\text{r}^2}{\text{r}} = \omega^2$r
2) For circular motion, $a = \omega^2 r$, so here we have $a = \omega^2 x_o$
3) The horizontal acceleration at x is $a = -a cos\theta$
4) Add 2 and 3 we get $a = -\omega^2 x_o cos\theta$

Defining Equation

1) The horizontal displacement at x is $x = x_o cos\theta$
2) Add 1 and acceleration and we get $a = -\omega^2 x$

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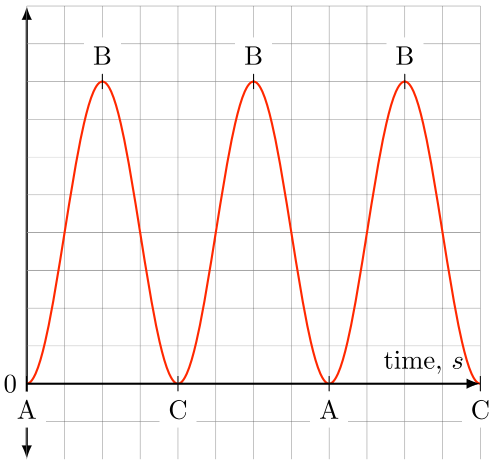
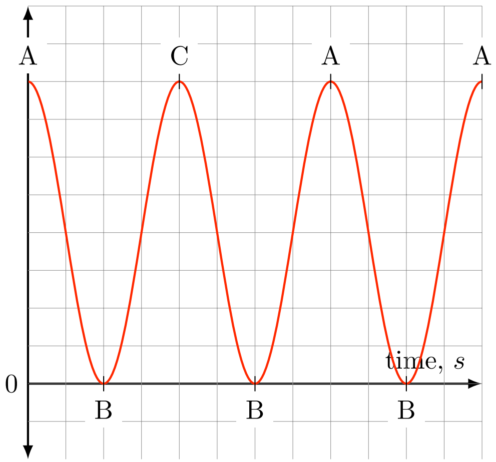
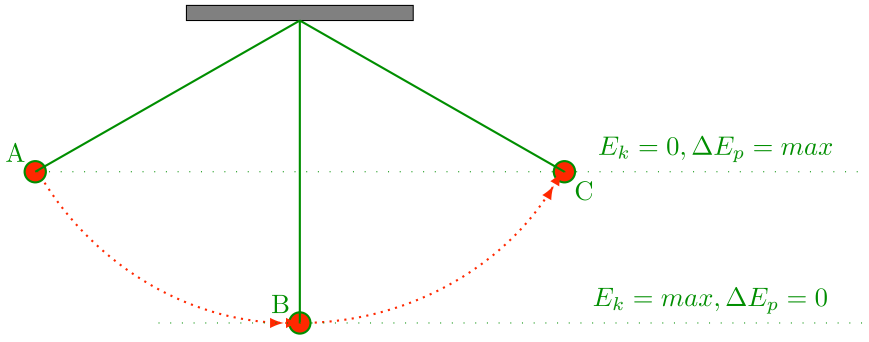
SHM Kinematics 3 (4.2.2)







Deriving E_k

- From Kinematics 1, $\mathbf{v = -v_o sin\omega t}$
- As $E_k = \frac{1}{2}mv^2$, E_k is now equal to

$$E_k = \frac{1}{2}mv_o^2 sin^2\omega t$$

Deriving ΔE_p

- We can deduce that ΔE_p has an opposite magnitude as E_k, but follows a cosine^2 curve
- As $E_k = \frac{1}{2}mv_o^2 sin^2\omega t$

$$\Delta E_p = \frac{1}{2}mv_o^2 cos^2\omega t$$

- Graphically you can see that E_k and ΔE_p add together to create a constant value, the *Total Mechanical Energy*
- This also parallels Pythagoras:
  $1 = cos^2\theta + sin^2\theta$

Deriving $\mathbf{v}_{max} = \omega\mathbf{x}$

$$1 = cos^2\theta + sin^2\theta$$
$$sin\theta = \sqrt{1 - cos^2\theta}$$
$$sin\omega t = \sqrt{1 - cos^2\omega t}$$
$$\omega x_o sin\omega t = \omega x_o\sqrt{1 - cos^2\omega t}$$
$$\omega x_o sin\omega t = \omega\sqrt{x_o^2 - x_o^2 cos^2\omega t}$$
$$x^2 = x_o^2 cos^2\omega t \quad \text{so}$$
$$v = \omega\sqrt{x_o^2 - x^2}$$

The maximum velocity is when displacement x=0 so therefore

$$v_{max} = \omega x$$

Deriving $E_{k,max}, \Delta E_p$

$$E_k = \frac{1}{2}mv^2$$
$$v = \omega\sqrt{x_o^2 - x^2}$$
$$E_k = \frac{1}{2}m\omega^2(x_o^2 - x^2)$$

The maximum E_k is when displacement x=0 so therefore

$$E_{k,max} = \frac{1}{2}m\omega^2(x_o^2)$$
$$\therefore \text{totalenergy} = \frac{1}{2}m\omega^2(x_o^2)$$

$$\Delta E_p = \text{totalenergy} - E_k$$
$$\Delta E_p = \frac{1}{2}m\omega^2(x_o^2) - \frac{1}{2}m\omega^2(x_o^2 - x^2)$$

$$\Delta E_p = \frac{1}{2}m\omega^2(x^2)$$